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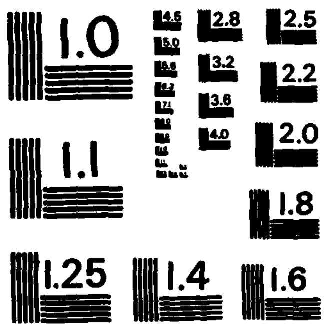

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The problem of assigning the closed loop poles of a linear time-invariant multivariable system using a proper, linear, time invariant, output feedback compensator continues to be of great interest. Even though several issues remain unresolved, good progress has been made, as evidenced by the interesting work of many researchers (see references) for a partial list).

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Reference 4

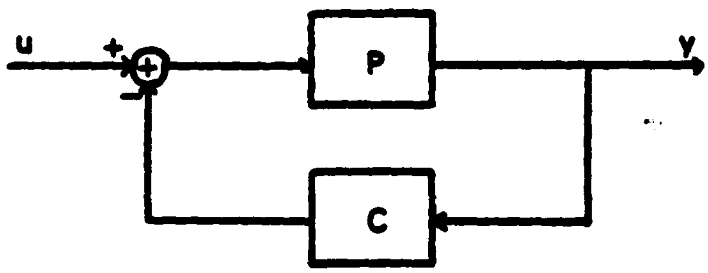
Another Approach to Generic Pole Assignment

AFOSR-80-0155

Detailed Summary

The problem of assigning the closed loop poles of a linear time-invariant multivariable system using a proper, linear, time invariant, output feedback compensator continues to be of great interest. Even though several issues remain unresolved, good progress has been made, as evidenced by the interesting work of many researchers (see references for a partial list).

Consider the following feedback configuration:



where P is a given strictly proper $m \times \ell$ transfer function (order n) and C an $\ell \times m$ proper transfer function (order q, which is to be constructed) both having elements in R(s) the field of rational functions in s over the reals R.

If one focuses attention on the constant (static) output feedback pole assignment problem, it is evident by counting dimensions that $m\ell \geq n$ is a necessary condition [14]. In a recent paper, Herman and Martin [8] show that $m\ell \geq n$ is a sufficient condition for generic pole assignment provided one allows complex matrices K in the feedback loop. Willems and Hesselink [14] show that for almost all systems with $m=\ell=2$, $n=4$, ($m\ell=n$) generic pole assignment (with real K) is <u>not</u> possible. On the other hand, Brockett and Byrnes [3] proceeding from a geometric viewpoint show that if either $\min(m,\ell)=1$ or $\min(m,\ell)=2$ and $\max(m,\ell)=2^k-1$, then $m\ell=n$ is a sufficient

condition for generic pole assignment. Using a different approach, Morse, Wolovich and Anderson [13] give a constructive proof of the fact that with $\ell=3$, $m=2$, $n=6$, $m\ell=n$ is a sufficient condition for generic pole assignment.

Perhaps the strongest result thus far is due to Kimura [10], (see also [1, 3, 5]). It states that for a controllable observable plant (order n, m outputs, ℓ inputs) it is "almost always" possible to assign $\min(n, m+\ell-1)$ closed loop poles arbitrarily close to a given set of real and complex conjugate values by constant output feedback. This implies that $m+\ell-1\geq n$ is a sufficient condition for generic pole assignment. In a follow-up paper, Kimura [11] gives a better bound $m+\ell+\lambda-1\geq n$, subject to the constraints that $m\geq\lambda$, $\ell\geq\mu$ where λ, μ are the controllability and observability indecies of the system, respectively.

The above results dealt with the question of pole assignment by constant output feedback (i.e. when the compensator was restricted to be of order zero). A very natural extension of these ideas is to consider the situation when a proper output feedback compensator of a fixed order q, is used. In 1970, Brasch and Pearson [2] showed that for a controllable observable plant, a compensator of order $q = \min(\lambda-1, \mu-1)$ is sufficient to achieve pole assignment. Recently, Willems and Hesselink [14] showed that $q(m+\ell-1) + m\ell\geq n$ is a necessary condition for generic pole assignment in the class of proper output feedback compensators of order q. Extending their constant output feedback result to the dynamic case Antsaklis and Wolovich [1] show with a compensator of order q, $\min(n+q, m+\ell+2q-1)$ closed loop poles generically assigned. This leads to a worse bound (in many cases) than the earlier Brasch and Pearson result [2]. Using a different approach [7], one can show that generically, $\min(n+q, (q+1)m+q)$ closed loop poles can be assigned with a compensator of order q. This is better than the Antsaklis-

Wolovich result[1] (in many cases) and coincides with the earlier Brasch and Pearson result [2], when $q = \nu-1$.

The approach suggested in [7] and employed in this paper as well, proceeds by using input-output transfer functions in the frequency domain and by exploiting a formulation based on matrix fraction descriptions [6, 9] and generalized Sylvester resultants [12].

Let the given system be expressed as:

$$P = N_{RP}D_{RP}^{-1},$$

and the feedback compensator C (to be found)

$$C = X^{-1}Y$$

where N_{RP}, D_{RP} are right coprime and X,Y left coprime. Then the closed loop transfer function is:

$$G = P(I + CP)^{-1},$$

and the closed loop characteristic polynomial [4]

$$\phi(s) = \alpha\det(\underbrace{XD_{RP} + YN_{RP}}_{\Phi}) \quad \alpha \neq 0 \text{ a constant.}$$

Now it can be shown [7] that if X, Y are restricted to be:

$$X = \begin{bmatrix} x(s), & 0, & \ldots, & 0 \\ 0 & & & \\ \vdots & & I_{\ell-1} & \\ 0 & & & \end{bmatrix} \qquad Y = \begin{bmatrix} y_{11}(s), & y_{12}(s), & \ldots, & y_{1\ell}(s), & \ldots, & y_{1m}(s) \\ 0 & & & & & \\ \vdots & & I_{\ell-1} & & & 0 \\ 0 & & & & & \end{bmatrix}$$

($x(s)$ a polynomial of degree q, $y_{1j}(s)$ of degree q) and if P has equal controllability indecies ($n=\lambda\ell$) which implies that N_{RP} D_{RP} can be written as

$$D_{RP} = Is^{\lambda} + D_{\lambda-1}s^{\lambda-1} + \ldots + D_0, \quad N_{RP} = N_{\lambda-1}s^{\lambda-1} + \ldots + N_0,$$

then $\phi(s)$ can be expressed as [7]

$$\phi(s) = x(s)\ \underbrace{(d_{11}(s)\ \Delta_{11}(s) + \ldots + d_{1\ell}\ \Delta_{1\ell}(s))}_{d}$$

$$+\ \underline{y}(s)\ \underbrace{(\underline{n}_1(s)\Delta_{11}(s) + \ldots + \underline{n}_\ell(s)\ \Delta_{1\ell}(s))}_{n}.$$

Now $\underline{y}(s)$ is the first row of Y, $(d_{11}(s), \ldots, d_{1\ell}(s))$ the first row of D_{hp}, $\underline{n}_j$ the j^{th} column of N_{hp}, $\Delta_{1j}(s)$ the appropriate $(\ell-1) \times (\ell-1)$ minors of ϕ as det is computed by expanding by the first row, (Δ_{1j} do not contain compensator parameters). Now $x(s)$, $\underline{y}(s)$ include $(q+1)m+q$ parameters, and $\phi(s)$ has coefficients which are linear in these parameters. This allows [7] for the generic arbitrary assignment of $(q+1)m+q$ closed loop poles.

Suppose now that the compensator structure is modified to become:

$$X = \begin{bmatrix} x(s), 0, \ldots, 0 \\ 0 & \\ \vdots & I_{\ell-1} \\ 0 & \end{bmatrix} \quad Y = \begin{bmatrix} y_{11}(s), & y_{12}(s), & \ldots, & y_{1\ell}(s), & \ldots, & y_{1m}(s) \\ e_1 & , e_2 & , & \cdots & & , e_m \\ f_1 & , f_2 & , & & & , f_m \\ & & & \vdots & & \\ h_1 & , h_2 & , & \cdots & & , h_m \end{bmatrix}$$

then ϕ will be of the form:

$$\phi = \begin{bmatrix} \phi_{11}, \phi_{12}, \ldots, \phi_{1\ell} \\ \phi_{21}, \phi_{22}, \ldots, \phi_{2\ell} \\ \vdots \\ \phi_{\ell 1}, \phi_{\ell 2}, \ldots, \phi_{\ell\ell} \end{bmatrix}$$

where ϕ_{1i} $1 \leq i \leq s$ will contain parameters from x, y,

ϕ_{2i} $1 \leq i \leq s$ will contain the parameters $g = (g_1, \ldots, g_m)$

ϕ_{3i} " $f = (f_1, \ldots, f_m)$

$\vdots$

ϕ_{ki} " $h = (h_1, \ldots, h_m)$

Now, if the g parameters are used so that ϕ_{2i} $1 \leq i \leq s$ have a common factor $h_1(s)$ then ϕ_{1j} $1 \leq j \leq s$ will also have this factor. Proceeding in a similar fashion for the other rows (3 to k) results in $\phi(s)$ being of the form:

$$\phi(s) = [x\,(d_{11}\hat{\Delta}_{11} + \cdots + d_{1s}\hat{\Delta}_{1s}) + y\,(g_1\hat{\Delta}_{11} + \cdots + g_s\hat{\Delta}_{1s})]\, h_1 h_2 \cdots h_{k-1}$$

The x, y can still be used to assign $(q-1)mq$ poles, which implies that potentially <u>more</u> than $(q-1)mq$ poles can be assigned. This idea does lead to improvements, as can be seen from the following two results.

<u>Definition</u>: A set $S \subseteq R^k$ is called <u>generic</u> if it contains a non-empty Zariski open set [15].

<u>Theorem 1.</u> Let $n=4$, $m=2$, $r=4$ and $P = N_{rp} D_{rp}^{-1}$ where

$$D_{rp} = Is^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0 = \begin{bmatrix} d_1(s) & d_2(s) \\ d_3(s) & d_4(s) \end{bmatrix}$$

$$N_{rp} = \qquad N_3 s^3 + N_2 s^2 + N_1 s + N_0 = \begin{bmatrix} n_1(s) & n_2(s) \\ n_3(s) & n_4(s) \\ n_5(s) & n_6(s) \\ n_7(s) & n_8(s) \end{bmatrix}$$

Let $k=6$, $\phi(s)$ the closed loop characteristic polynomial,

$W = \{(N_{rp}, D_{rp}) \in R^{(m+r)n} \mid N_{rp}, D_{rp} \text{ as above}\}$,

$S = \{\underline{s} = (s_1, s_2, \ldots, s_k) \in R^k \mid s_i \text{ real}\}$,

$Z = \{\underline{z} = (N_{rp}, D_{rp}, \underline{s}) \in R^{(m+r)n} \times R^k \mid$ for which there exists a constant compensator such that $s_1, \ldots, s_k$ are roots of $\phi(s)\}$

Then Z is a generic subset of $R^{(m+r)n} \times R^k$.

The theorem suggests that for almost all 4x2 transfer functions of McMillan degree 8 (and equal controllability indecies) and almost all $\underline{s} = (s_1, \ldots, s_6)$ s_i real there exists a constant compensator which assigns 6 poles. This result is better than the result in [7]. It is also better than $m+p-1 = 5$, which is perhaps the best result known to date [10, 11].

__Theorem 2.__ Let $m=4$, $p=2$, $n=10$ and $P = N_{rp} D_{rp}^{-1}$ where

$$D_{rp} = Is^5 + D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0 = \begin{bmatrix} d_1(s) & d_2(s) \\ d_3(s) & d_4(s) \end{bmatrix}$$

$$N_{rp} = \qquad N_4 s^4 + N_3 s^3 + N_2 s^2 + N_1 s + N_0 = \begin{bmatrix} n_1(s) & n_2(s) \\ n_3(s) & n_4(s) \\ n_5(s) & n_6(s) \\ n_7(s) & n_8(s) \end{bmatrix}$$

Let $k=11$, $\phi(s)$ the closed loop characteristic polynomial.

$$U = \{(N_{rp}, D_{rp}) \in R^{(m+p)n} \mid N_{rp}, D_{rp} \text{ as above}\}$$

$$S = \{\underline{s} = (s_1, \ldots, s_k) \in R^k \mid s_i \text{ real}\}$$

$$Z = \left\{\underline{s} = (N_{rp}, D_{rp}, \underline{s}) \in R^{(m+p)n} \times R^k \;\middle|\; \begin{array}{l} \text{for which there exists a proper} \\ \text{compensator of order 1 such that} \\ s_1, \ldots, s_k \text{ are roots of } \phi(s). \end{array}\right\}$$

Then Z is a generic subset of $R^{(m+p)n} \times R^k$.

The theorem suggests that for almost all 4x2 transfer functions of McMillan degree 10 (and equal controllability indecies) and almost all $\underline{s} = (s_1, \ldots, s_{11})$ s_i real there exists a proper compensator of order 1 which assigns <u>all</u> 11 poles. This result is better than the result in [7]. It is also better than the best known dynamic output feedback result [2] which when applied to this case would require a compensator of order $\frac{n}{4} - 1 = \frac{3}{2}$ therefore of order 2. It is also interesting to note that if we apply the

necessary condition of Willems and Hesselink [14] we find that

$$q(m+q-1) \geq mqn$$

or

$$q \geq \frac{10-8}{5} = \frac{2}{5} \quad \text{i.e.} \quad q \geq 1.$$

which implies that this is the best we could possibly do.

The above preliminary results are very encouraging. Work along these lines is therefore continuing and more general theorems are being formulated. Complex poles are treated in the same manner.

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